Limits at Infinity When Variable is Exponent

Sample Problems

a) $\lim_{x\to\infty} 2^x$ and b) $\lim_{x\to-\infty} 2^x$

Solution: Since $2>1$, these limits are ∞ and 0, i.e. $\lim_{x\to\infty} 2^x = \infty$ and $\lim_{x\to-\infty} 2^x = 0$.

c) $\lim_{x\to\infty}\left(\frac{2}{3}\right)^x$ and d) $\lim_{x\to-\infty}\left(\frac{2}{3}\right)^x$

Solution: Since $\frac{2}{3}<1$, these limits are 0 and ∞, i.e. $\lim_{x\to\infty}\left(\frac{2}{3}\right)^x = 0$ and $\lim_{x\to-\infty}\left(\frac{2}{3}\right)^x = \infty$.

e) $\lim_{x\to\infty}\frac{2^{x+3}}{3^{x+1}}$

Solution: We start by re-writing the exponential expressions. The goal is to bring it into a form where there is only one exponential expression involving x.

$$\frac{2^{x+3}}{3^{x+1}} = \frac{2^x\cdot 2^3}{3^x\cdot 3^1} = \frac{2^x\cdot 8}{3^x\cdot 3} = \frac{8}{3}\left(\frac{2}{3}\right)^x$$

$$\text{Thus}\quad \lim_{x\to\infty}\frac{2^{x+3}}{3^{x+1}} = \lim_{x\to\infty}\frac{8}{3}\left(\frac{2}{3}\right)^x = \frac{8}{3}\lim_{x\to\infty}\left(\frac{2}{3}\right)^x = 0 \qquad \text{since } \frac{2}{3}<1$$

f) $\lim_{x\to-\infty}\frac{2^{x+3}}{3^{x+1}}$

Solution: $\lim_{x\to-\infty}\frac{2^{x+3}}{3^{x+1}} = \lim_{x\to-\infty}\frac{2^{x+3}}{3^{x+1}} = \lim_{x\to-\infty}\frac{8}{3}\left(\frac{2}{3}\right)^x = \frac{8}{3}\lim_{x\to-\infty}\left(\frac{2}{3}\right)^x = \infty$

g) $\lim_{x\to\infty}\frac{2^{2x+1}}{3^{x-1}}$

Solution: We start by re-writing the exponential expressions. The goal is to bring it into a form where there is only one exponential expression involving x.

$$\frac{2^{2x+1}}{3^{x-1}} = \frac{2^{2x}\cdot 2^1}{\frac{3^x}{3^1}} = \frac{(2^2)^x\cdot 2}{3^x\cdot\frac{1}{3}} = \frac{4^x\cdot 6}{3^x} = 6\left(\frac{4}{3}\right)^x$$

$$\text{Thus}\quad \lim_{x\to\infty}\frac{2^{2x+1}}{3^{x-1}} = \lim_{x\to\infty} 6\left(\frac{4}{3}\right)^x = 6\lim_{x\to\infty}\left(\frac{4}{3}\right)^x = \infty \quad \text{since } \frac{4}{3}>1$$

h) $\lim_{x\to-\infty}\frac{2^{2x+1}}{3^{x-1}}$

Solution: $\lim_{x\to-\infty}\frac{2^{2x+1}}{3^{x-1}} = \lim_{x\to-\infty} 6\left(\frac{4}{3}\right)^x = 6\lim_{x\to-\infty}\left(\frac{4}{3}\right)^x = 0$

3. Compute each of the following limits.

a) $\lim_{x\to\infty}\frac{1}{x}$

Solution: This is a very important limit. Since the limit we are asked for is as x approaches infinity, we should think of x as a very large positive number. The reciprocal of a very large positive number is a very small positive number. This limit is 0.

b) $\lim_{x\to-\infty}\frac{1}{x}$

Solution: Since the limit we are asked for is as x approaches negative infinity, we should think of x as a very large negative number. The reciprocal of a very large negative number is a very small negative number. This limit is 0.