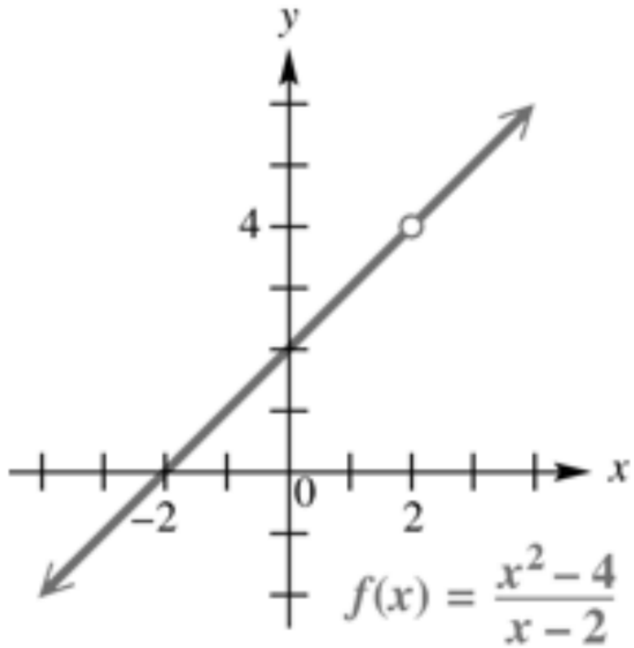
Unit 1: Limits

- **An Introduction To Limits**
- Techniques for Calculating Limits
- One-Sided Limits; Limits Involving Infinity

Limit of a Function

The function

$$f(x)=\frac{x^2-4}{x-2}$$



is not defined at $x = 2$, so its graph has a "hole" at $x = 2$.

Limit of a Function

Values of $f(x) = \dfrac{x^2 - 4}{x - 2}$ may be computed near $x = 2$

x approaches 2

❹

x	1.9	1.99	1.999→	←2.001	2.01	2.1
$f(x)$	3.9	3.99	3.999→	←4.001	4.01	4.1

❷

$f(x)$ approaches 4

Limit of a Function

The values of $f(x)$ get closer and closer to 4 as x gets closer and closer to 2.

We say that

"the limit of $\dfrac{x^2-4}{x-2}$ as x approaches 2 equals 4"

and write

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=4.$$

Limit of a Function

Limit of a Function

Let f be a function and let a and L be real numbers.

L* is the limit of *f*(*x*) as *x* approaches *a, written

$$\lim_{x \to a} f(x) = L,$$

if the following conditions are met.

1. As x assumes values closer and closer (but not equal) to a on both sides of a, the corresponding values of $f(x)$ get closer and closer (and are perhaps equal) to L.
2. The value of $f(x)$ can be made as close to L as desired by taking values of x arbitrarily close to a.

Finding the Limit of a Polynomial Function

Example Find $\lim_{x \to 1} (x^2 - 3x + 4)$.

Solution The behavior of $f(x) = x^2 - 3x + 4$ near $x = 1$ can be determined from a table of values,

x approaches 1

❹

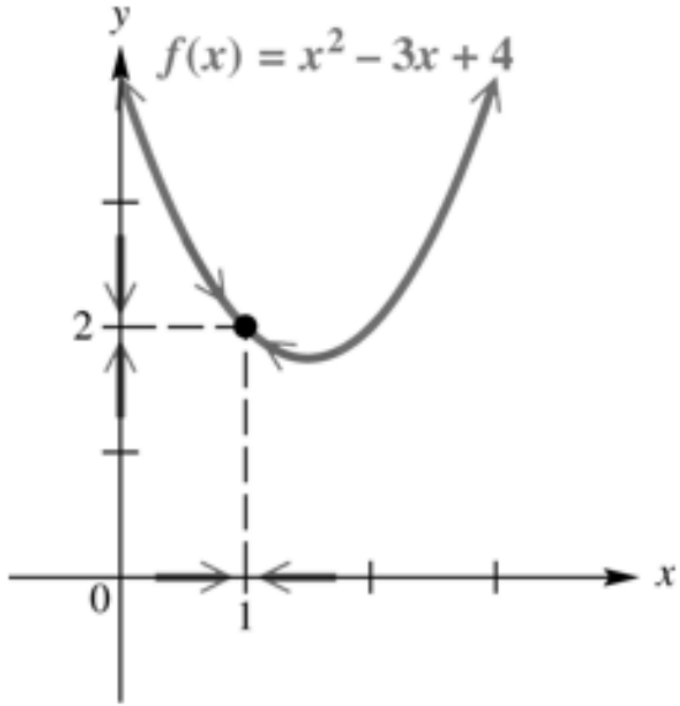
x	.9	.99	.999→	←1.001	1.01	1.1
$f(x)$	2.11	2.0101	2.001→	←1.999	1.9901	1.91

❷

$f(x)$ approaches 2

Finding the Limit of a Polynomial Function

Solution or from a graph of $f(x)$.



We see that

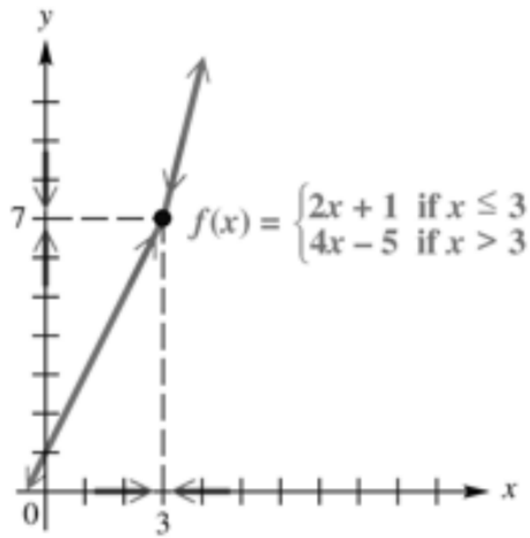
$$\lim_{x \to 1} (x^2 - 3x + 4) = 2.$$

Finding the Limit of a Polynomial Function

Example Find $\lim_{x \to 3} f(x)$ where

$$f(x) = \begin{cases} 2x+1 & \text{if } x \le 3 \\ 4x-5 & \text{if } x > 3 \end{cases}.$$

Solution Create a graph and table.



Finding the Limit of a Polynomial Function

Solution

x approaches 3

❹

x	2.9	2.99	2.999→	←3.001	3.01	3.1
$f(x)$	6.8	6.98	6.998→	←7.004	7.04	7.4

❷

$f(x)$ approaches 7

Therefore $\lim_{x \to 3} f(x) = 7$.

Limits That Do Not Exist

If there is no single value that is approached by $f(x)$ as x approaches a, we say that $f(x)$ does not have a limit as x approaches a, or $\lim_{x \to 2} f(x)$ does not exist.

Determining Whether a Limit Exists

Example Find $\lim_{x \to 2} f(x)$ where

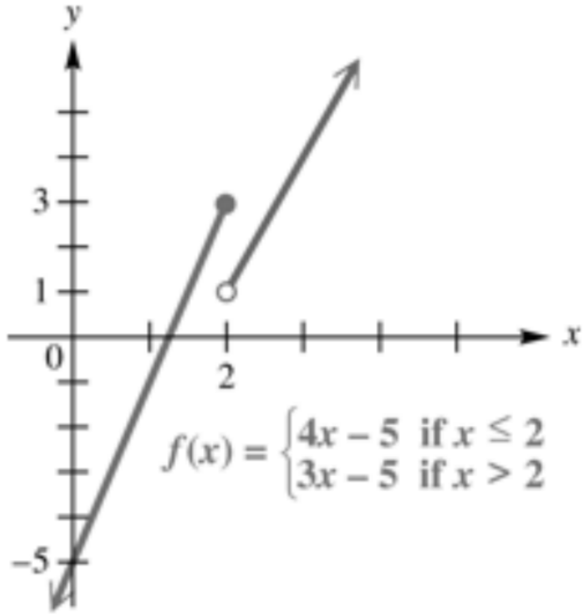
$$f(x) = \begin{cases} 4x - 5 & \text{if } x \le 2 \\ 3x - 5 & \text{if } x > 2 \end{cases}.$$

Solution Construct a table and graph

x	1.9	1.99	1.999→	←2.001	2.01	2.1
$f(x)$	2.6	2.96	2.996→	←1.003	1.03	1.3

Determining Whether a Limit Exists

Solution



$f(x)$ approaches 3 as x gets closer to 2 from the left,
$f(x)$ approaches 1 as x gets closer to 2 from the right.

Therefore, $\lim_{x \to 2} f(x)$ does not exist.

Determining Whether a Limit Exists

Example Find $\lim_{x \to 0} f(x)$ where $f(x) = \frac{1}{x^2}$.

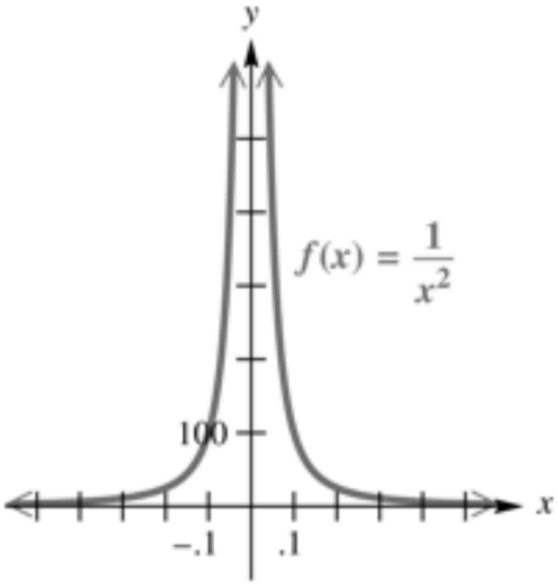
Solution Construct a table and graph

x	-.1	-.01	-.001→
$f(x)$	100	10,000	1,000,000→

x	←.001	.01	.1
$f(x)$	←1,000,000	10,000	100

Determining Whether a Limit Exists

Solution



As x approaches 0, the corresponding values of $f(x)$ grow arbitrarily large.

Therefore, $\lim_{x \to 0} \frac{1}{x^2}$ does not exist.

Limit of a Function

Conditions Under Which $\lim_{x \to a} f(x)$ Fails To Exist

1. $f(x)$ approaches a number L as x approaches a from the left and $f(x)$ approaches a different number M as x approaches a from the right.

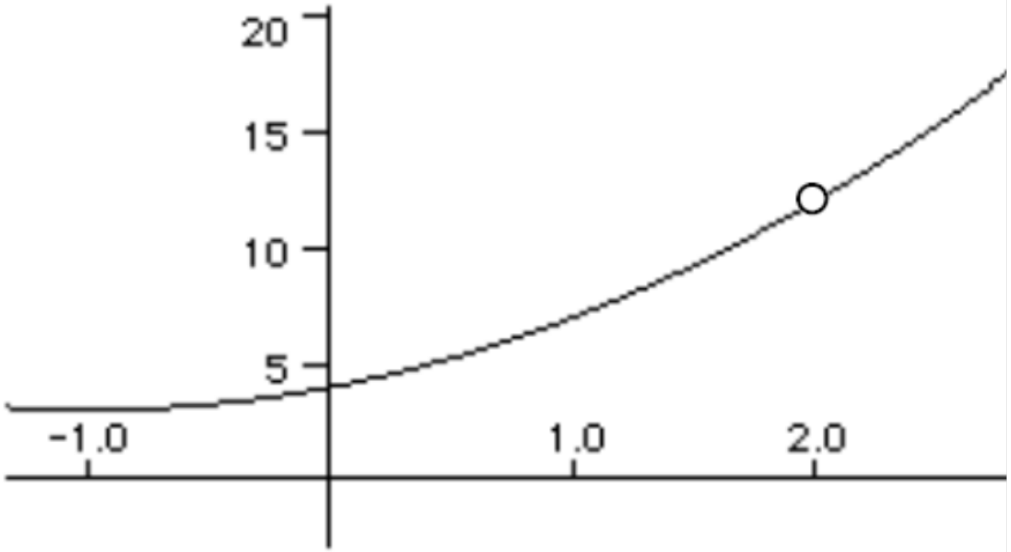
2. $f(x)$ becomes infinitely large in absolute value as x approaches a from either side.

3. $f(x)$ oscillates infinitely many times between two fixed values as x approaches a.

pose you were to graph

$$) = \frac{x^3 - 8}{x - 2}, \quad x \neq 2$$

all values of x not equal to 2,
can use standard curve sketching
niques. But the curve is not defined at
2 . There is a hole in the graph. So let's
an idea of the behavior of the curve around
2 .



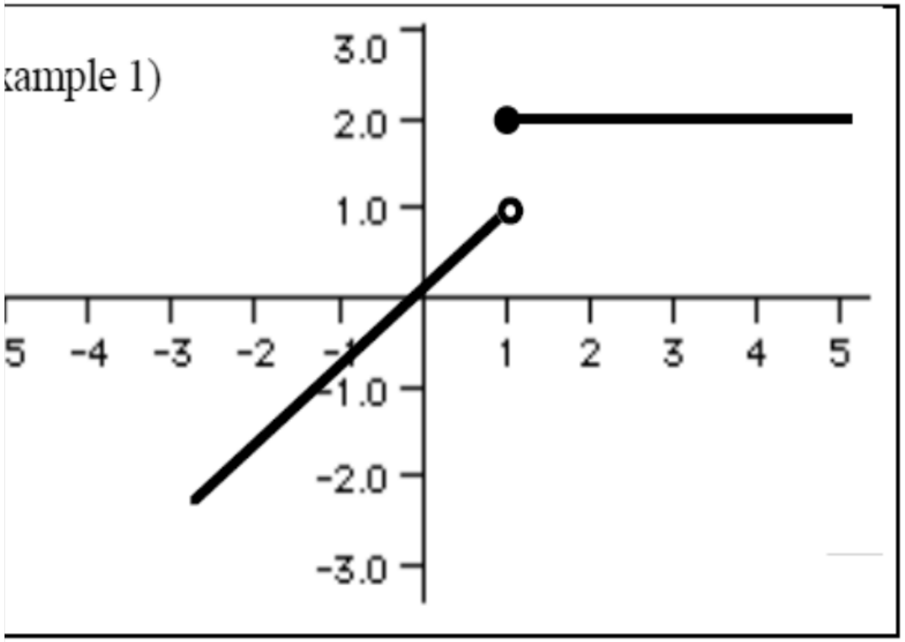
r calculator to 4 decimal accuracy and complete the chart.

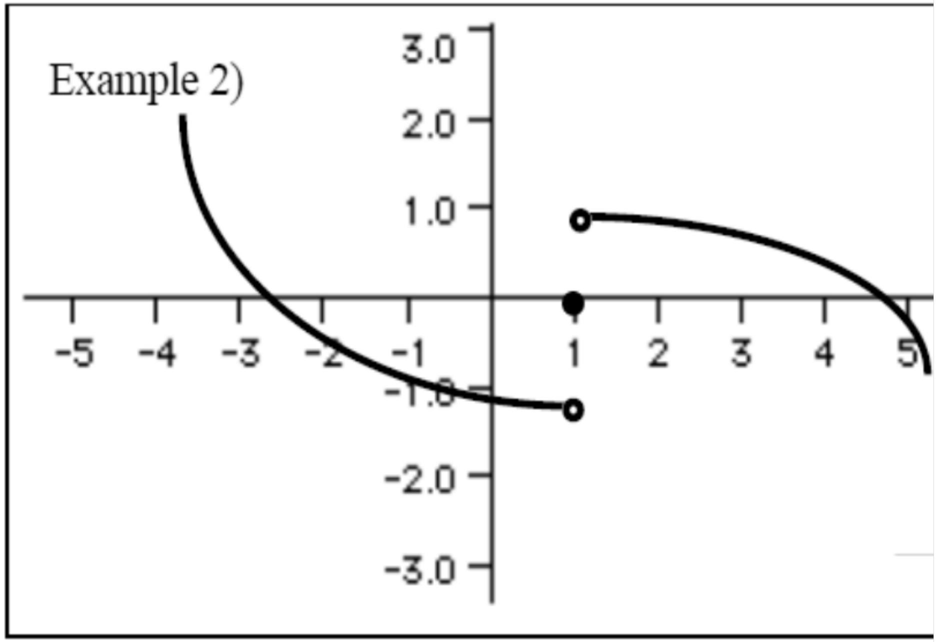
	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2

ld be obvious that as x gets closer and closer to 2, the value of $f(x)$ becomes closer and closer to e want the limit of $f(x)$ as we approach some value of c from the left hand side, we will write $\lim\limits_{x\to c^-} f(x)$.

e want the limit of $f(x)$ as we approach some value of c from the right hand side, we will write $\lim\limits_{x\to c^+} f(x)$.

rder for a limit to exist at c, $\lim\limits_{x\to c^-} f(x)$ must equal $\lim\limits_{x\to c^+} f(x)$ and we say $\lim\limits_{x\to c} f(x) = L$.



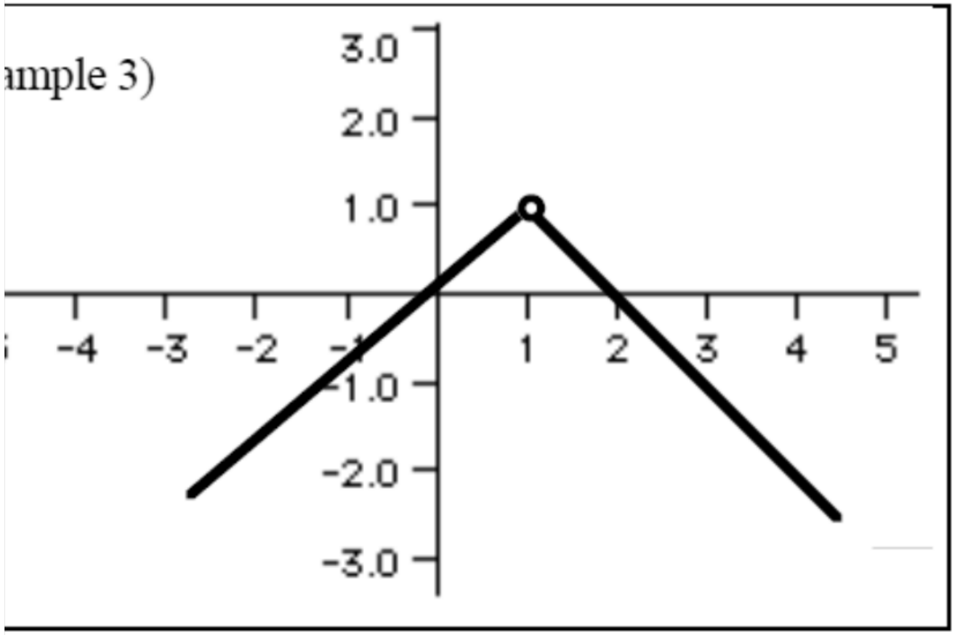


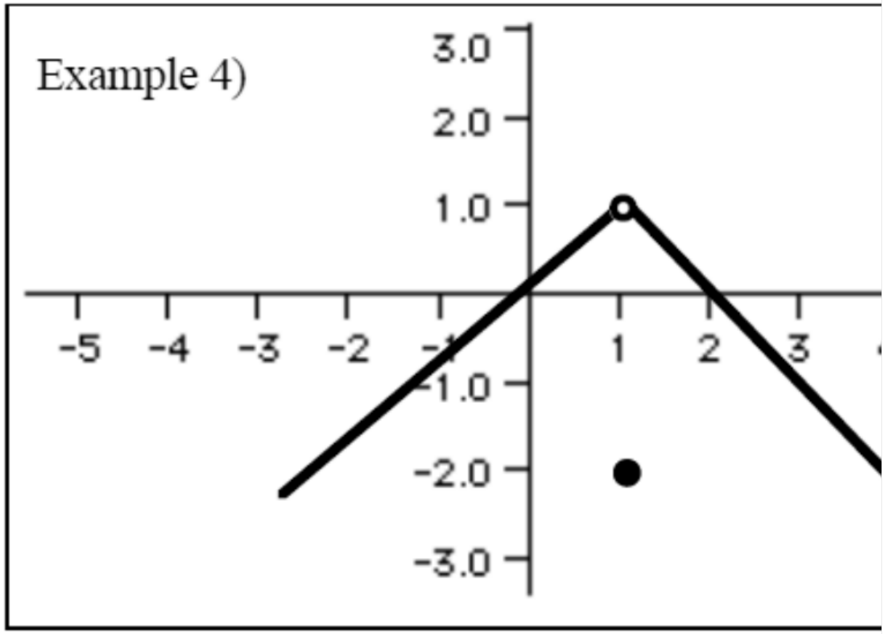
$\lim\limits_{x\to 1^-} f(x) =$ ______ $\lim\limits_{x\to 1^+} f(x) =$ ______

$\lim\limits_{x\to 1} f(x) =$ ______ $f(1) =$ ______

$\lim\limits_{x\to 1^-} f(x) =$ ______ $\lim\limits_{x\to 1^+} f(x) =$ ______

$\lim\limits_{x\to 1} f(x) =$ ______ $f(1) =$ ______





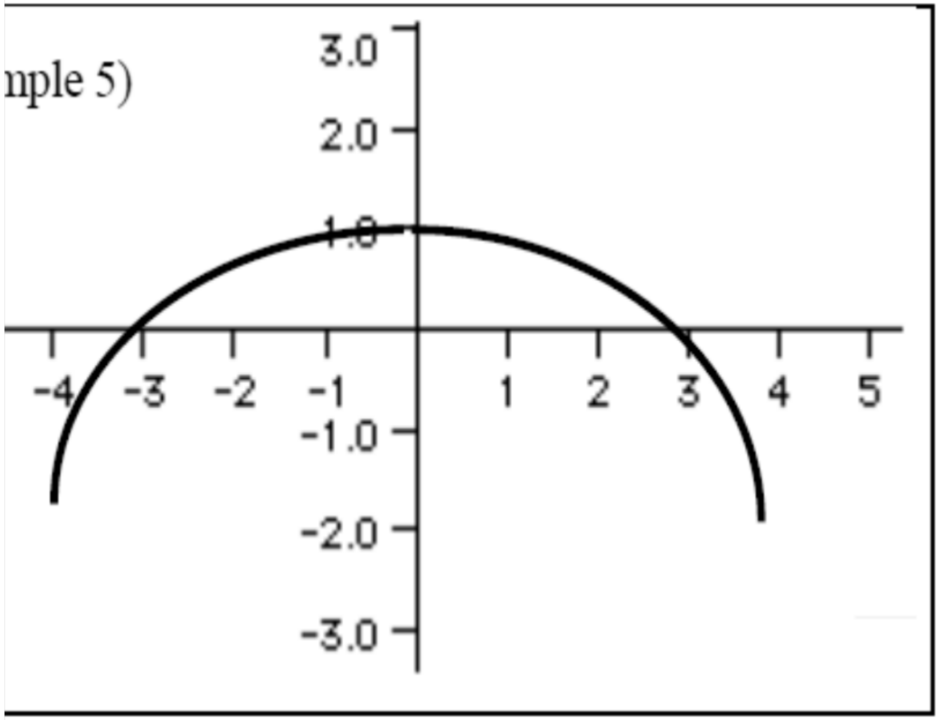
$\lim_{x\to 1^-} f(x) =$ ______ $\lim_{x\to 1^+} f(x) =$ ______

$\lim_{x\to 1} f(x) =$ ______ $f(1) =$ ______

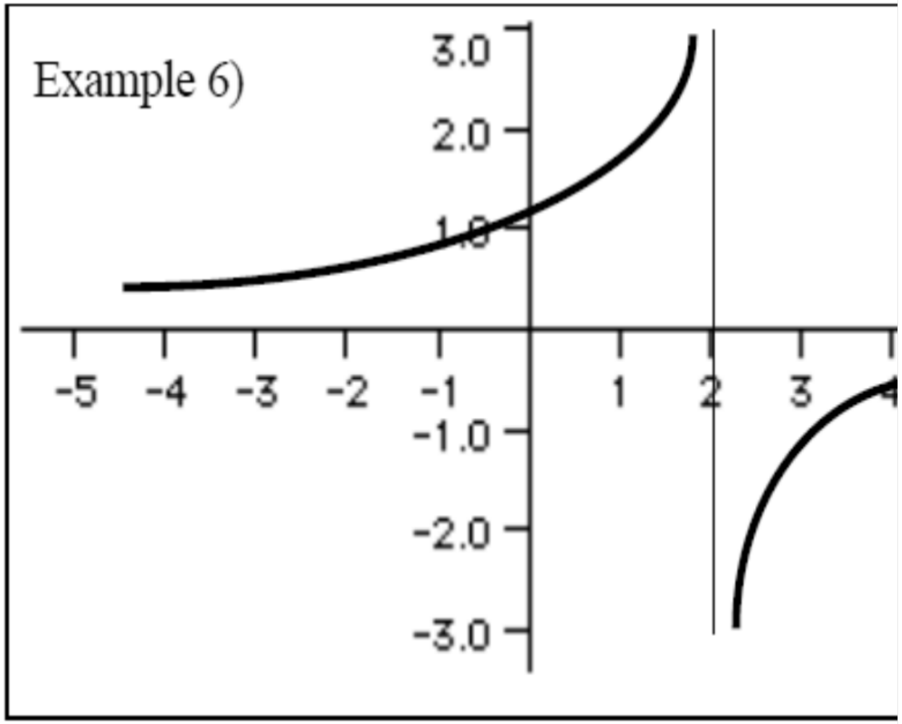
$\lim_{x\to 1^-} f(x) =$ ______ $\lim_{x\to 1^+} f(x) =$ ______

$\lim_{x\to 1} f(x) =$ ______ $f(1) =$ ______

nple 5)



Example 6)

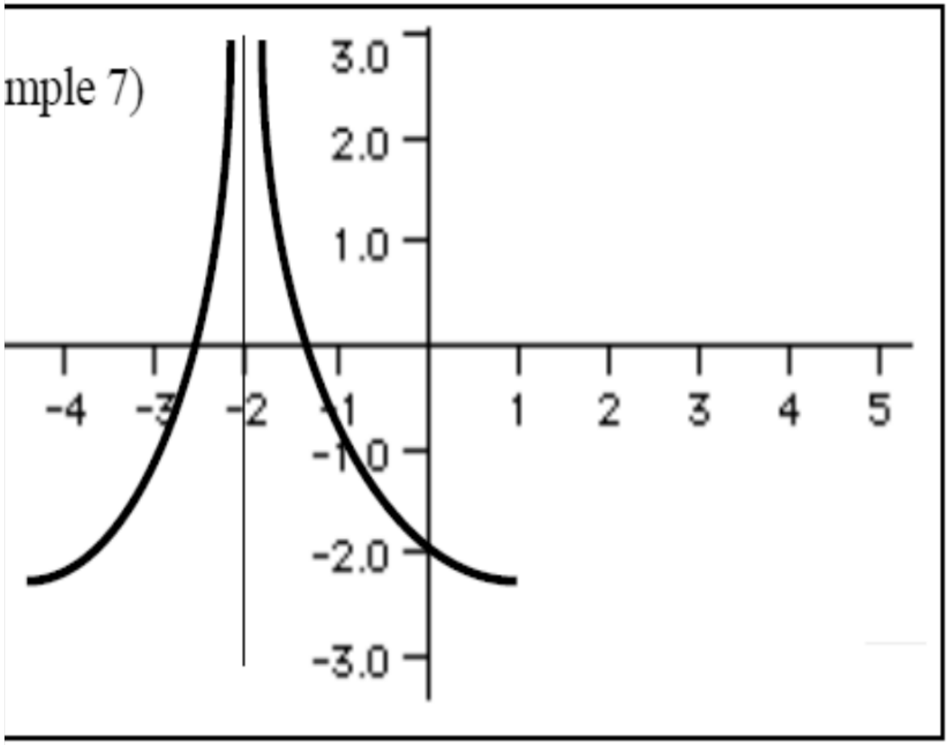


$(x) =$ ______ $\lim_{x \to 0^+} f(x) =$ ______

$(x) =$ ______ $f(0) =$ ______

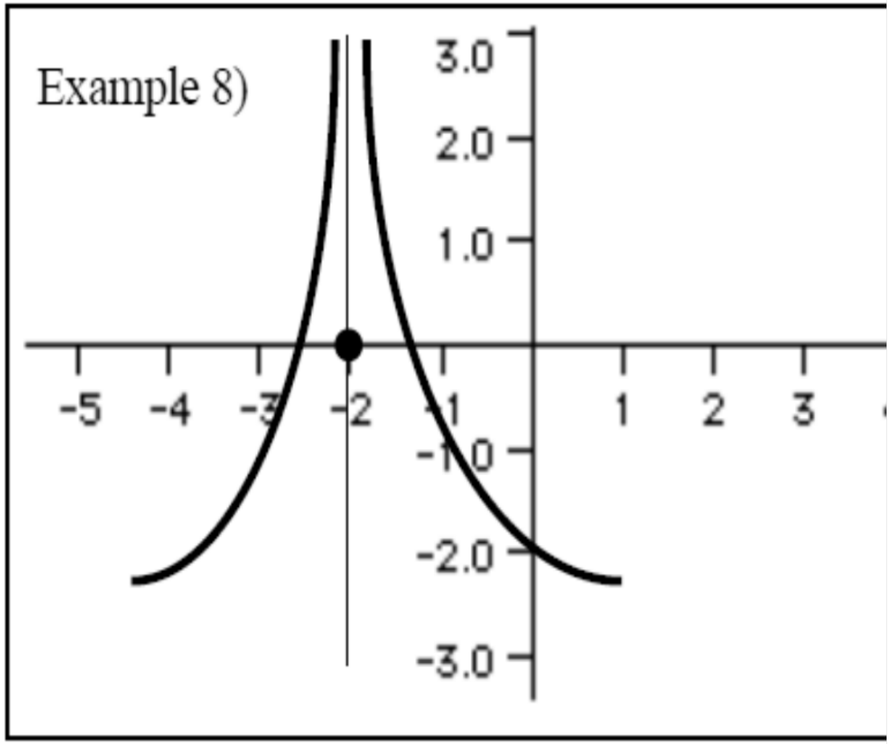
$\lim_{x \to 2^-} f(x) =$ ______ $\lim_{x \to 2^+} f(x) =$ ______

$\lim_{x \to 2} f(x) =$ ______ $f(2) =$ ______



$\text{n}_{-2^-} f(x) =$ ______ $\lim_{x \to -2^+} f(x) =$ ______

$\text{n}_{-2} f(x) =$ ______ $f(-2) =$ ______



$\lim_{x \to -2^-} f(x) =$ ______ $\lim_{x \to -2^+} f(x) =$ ______

$\lim_{x \to -2} f(x) =$ ______ $f(-2) =$ ______