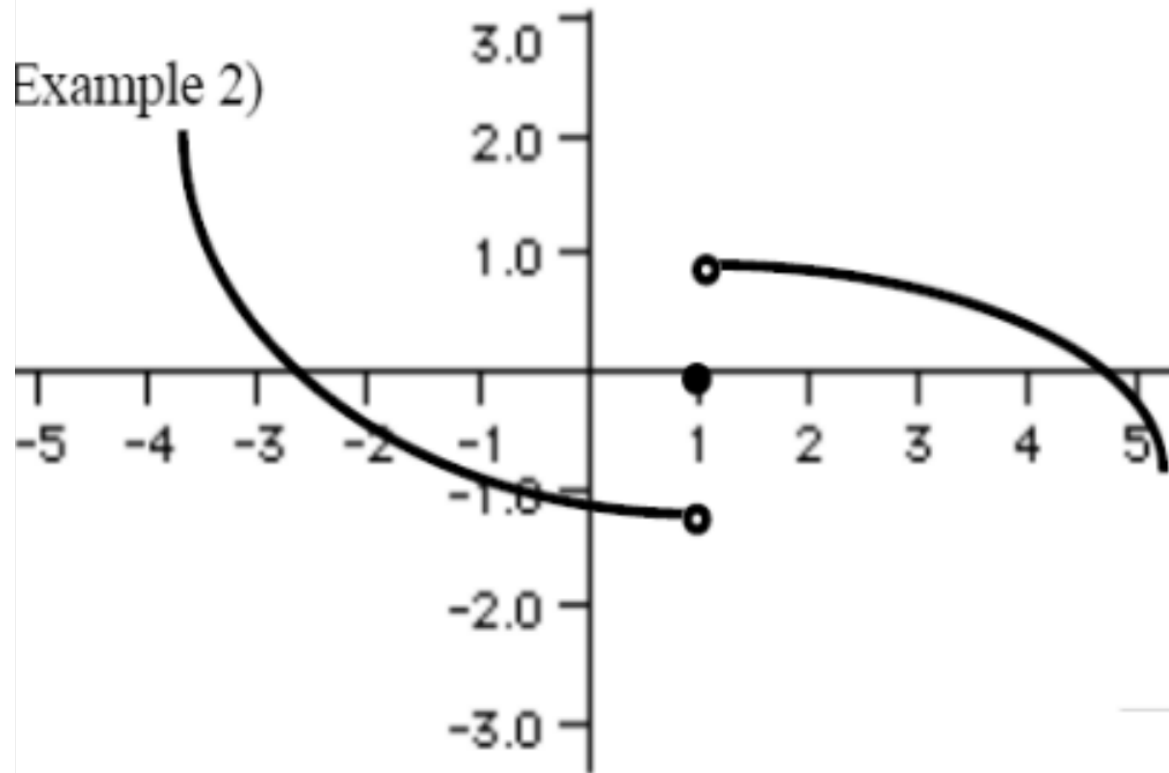


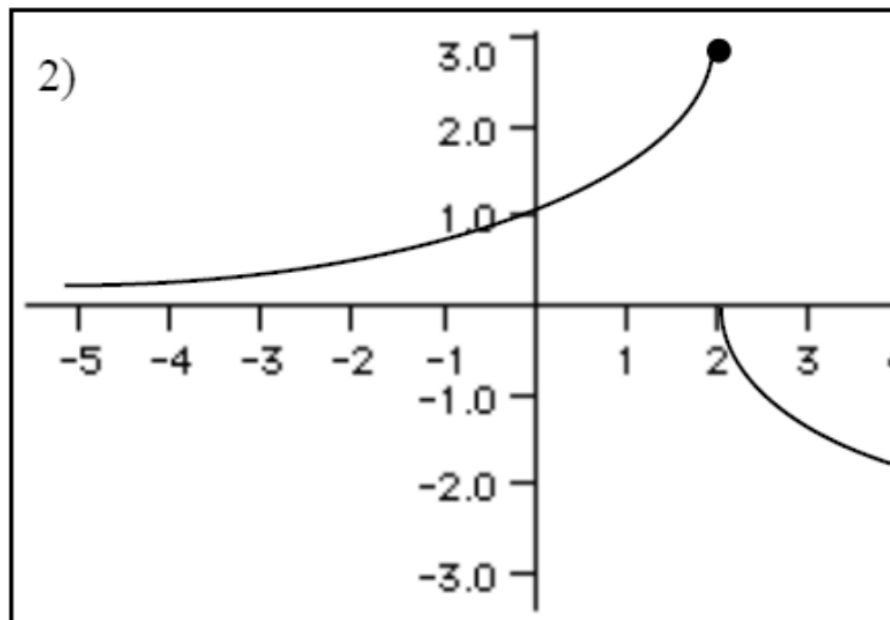
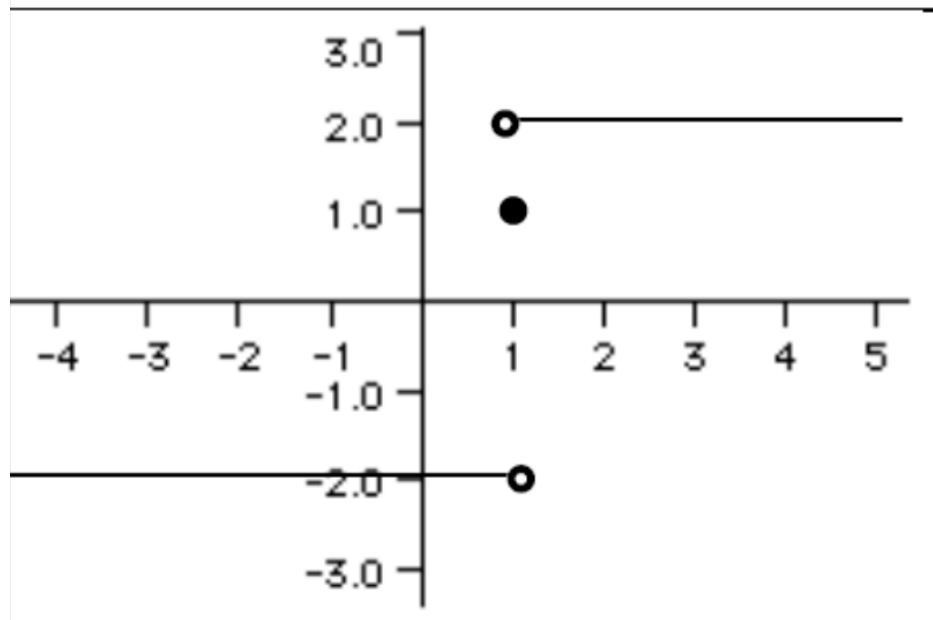
Warmup



Explain why the limit does not exist.

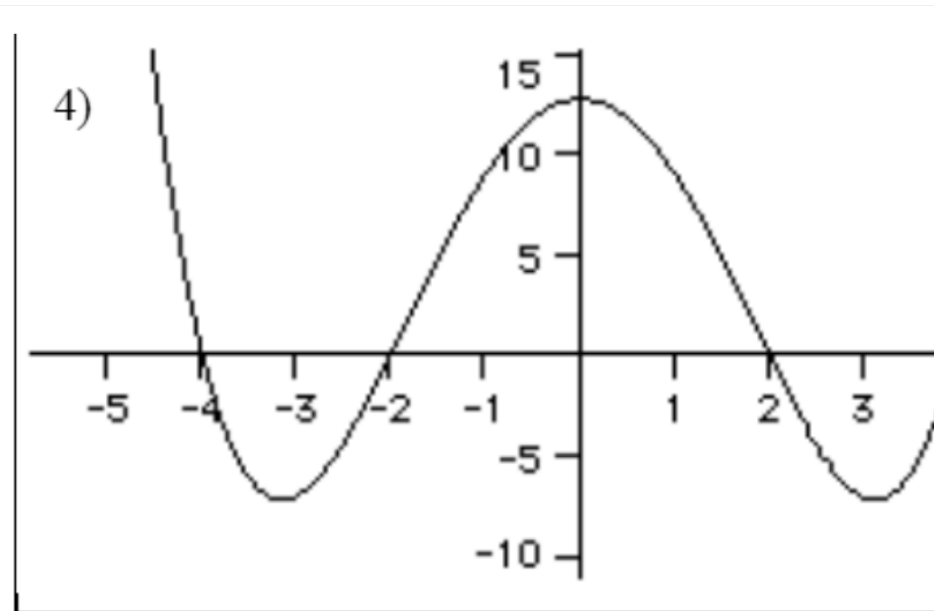
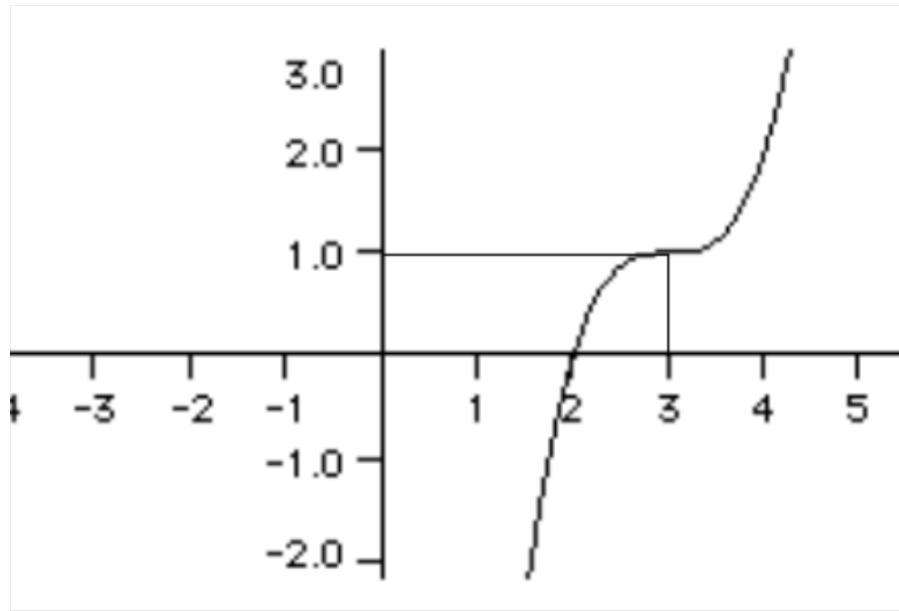
$$\lim_{x \rightarrow 1^-} f(x) = \text{---} \quad \lim_{x \rightarrow 1^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 1} f(x) = \text{---} \quad f(1) = \text{---}$$



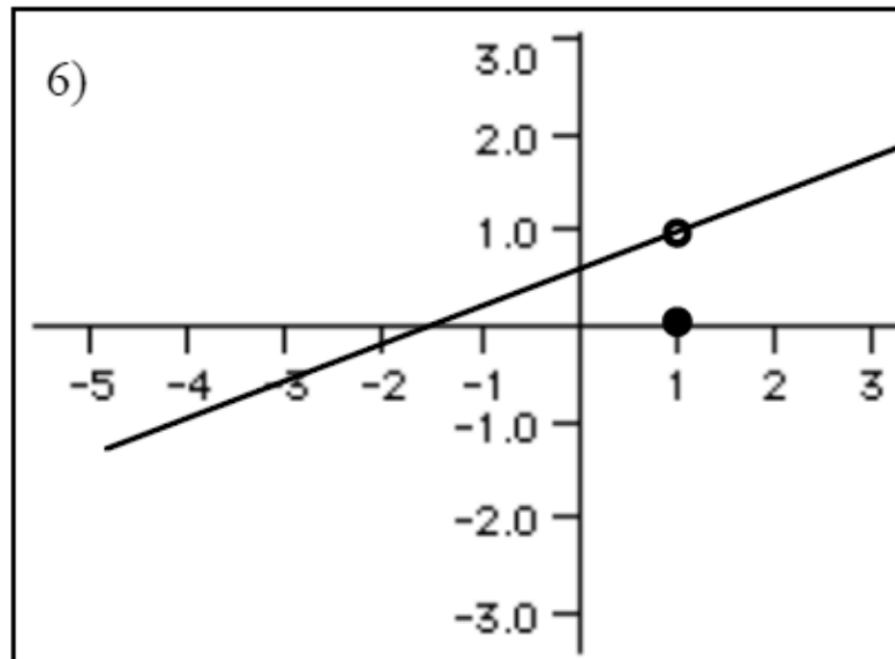
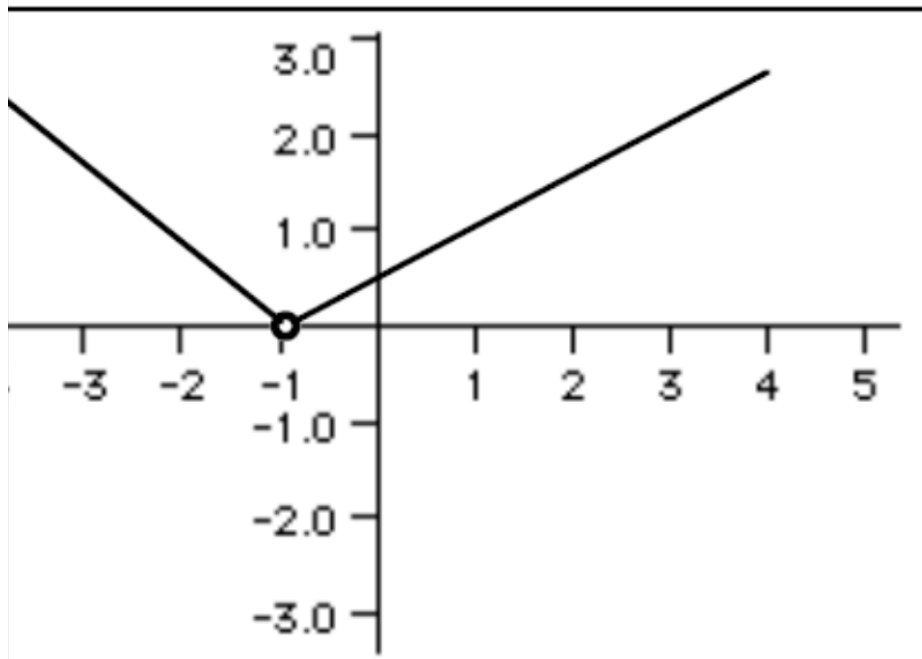
- $\lim_{x \rightarrow 1^-} f(x)$ b) $\lim_{x \rightarrow 1^+} f(x)$ c) $\lim_{x \rightarrow 1} f(x)$
 1) e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$

- a) $\lim_{x \rightarrow 2^-} f(x)$ b) $\lim_{x \rightarrow 2^+} f(x)$ c) $\lim_{x \rightarrow 2} f(x)$
 d) $f(2)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



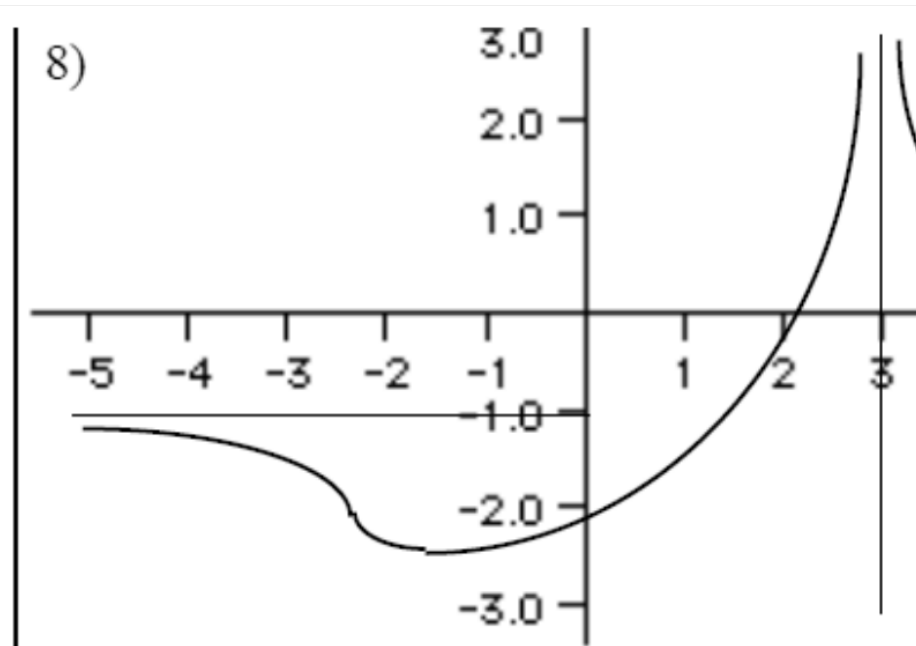
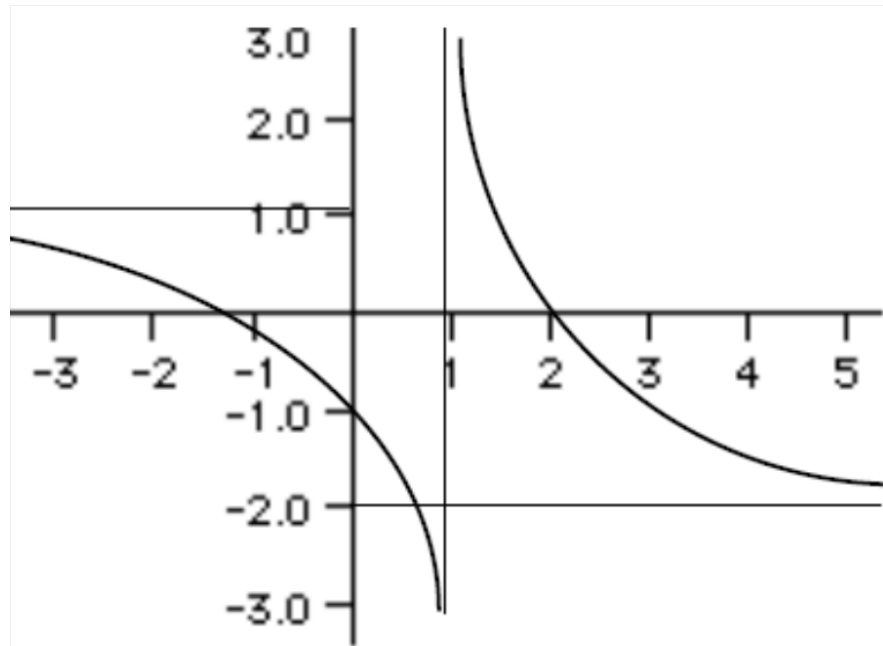
- $f(x)$ b) $\lim_{x \rightarrow 3^+} f(x)$ c) $\lim_{x \rightarrow 3} f(x)$
 e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$

- a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
 d) $f(0)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



- (x) b) $\lim_{x \rightarrow -1^+} f(x)$ c) $\lim_{x \rightarrow -1} f(x)$
 e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$

- a) $\lim_{x \rightarrow 1^-} f(x)$ b) $\lim_{x \rightarrow 1^+} f(x)$ c) $\lim_{x \rightarrow 1} f(x)$
 d) $f(1)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



$f(x)$

b) $\lim_{x \rightarrow 1^+} f(x)$

c) $\lim_{x \rightarrow 1} f(x)$

a) $\lim_{x \rightarrow 3^-} f(x)$

b) $\lim_{x \rightarrow 3^+} f(x)$

c) $\lim_{x \rightarrow 3} f(x)$

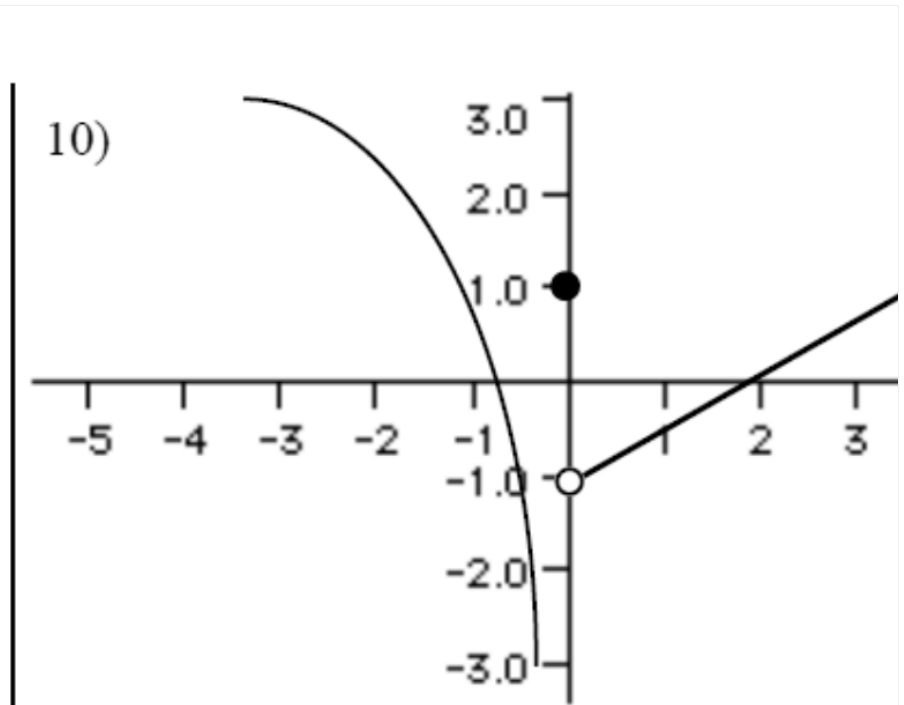
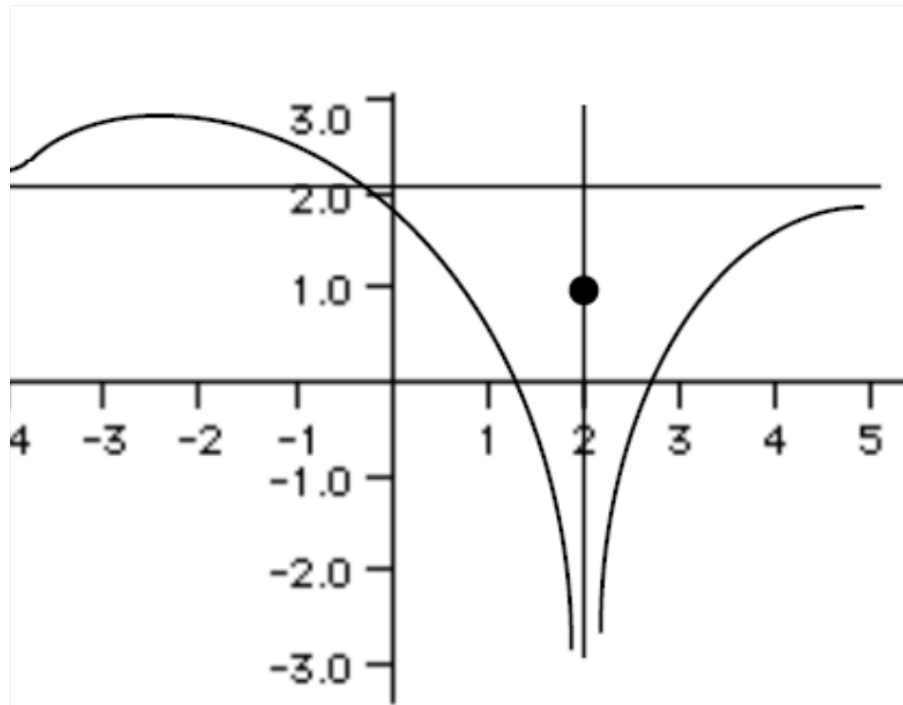
e) $\lim_{x \rightarrow -\infty} f(x)$

f) $\lim_{x \rightarrow \infty} f(x)$

d) $f(3)$

e) $\lim_{x \rightarrow -\infty} f(x)$

f) $\lim_{x \rightarrow \infty} f(x)$



$$f(x)$$

b) $\lim_{x \rightarrow 2^+} f(x)$

c) $\lim_{x \rightarrow 2} f(x)$

a) $\lim_{x \rightarrow 0^-} f(x)$

b) $\lim_{x \rightarrow 0^+} f(x)$

c) $\lim_{x \rightarrow 0} f(x)$

d)

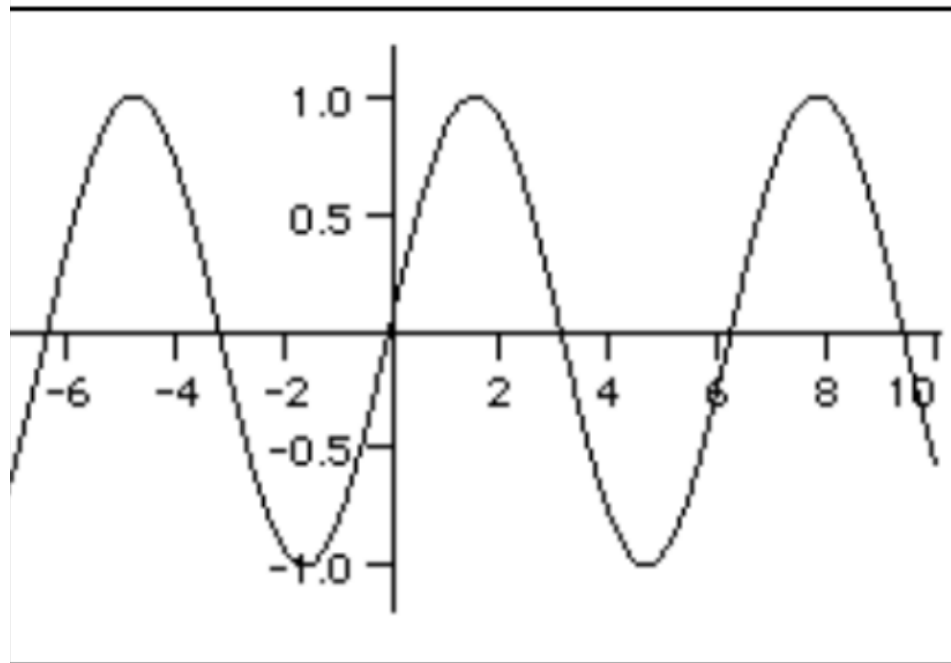
e) $\lim_{x \rightarrow -\infty} f(x)$

f) $\lim_{x \rightarrow \infty} f(x)$

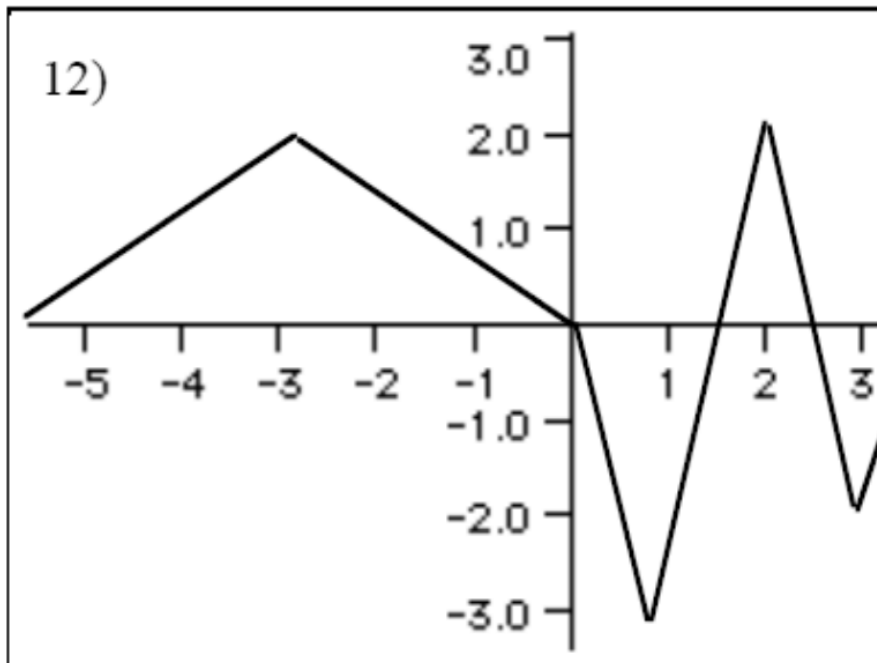
d) $f(0)$

e) $\lim_{x \rightarrow -\infty} f(x)$

f) $\lim_{x \rightarrow \infty} f(x)$



- $f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
 e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



- a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
 d) $f(0)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$

Strategies for Finding Limits

-Factor and Cancel

-Multiply by Conjugate

-Get Common Denominator

Factor and Cancel

Evaluate $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$.

Try plugging 5 into x — you should *always* try substitution first.

You get $\frac{0}{0}$ — no good, on to plan B.

Factor:

$$\begin{aligned} & \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} \\ &= \lim_{x \rightarrow 5} \frac{(x - 5)(x + 5)}{x - 5} \end{aligned}$$

3. Cancel the $(x - 5)$ from the numerator and denominator:

$$= \lim_{x \rightarrow 5} (x + 5)$$

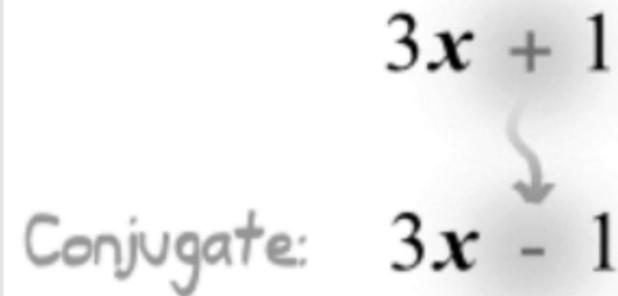
4. Now substitution will work.

You try!

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

Multiply by Conjugate

The conjugate is where we change the sign in the middle of 2 terms like this:


$$\begin{array}{l} 3x + 1 \\ \text{Conjugate: } 3x - 1 \end{array}$$

is an example where it will help us find a limit:

$$\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{4 - x}$$

Evaluating this at $x=4$ gives $0/0$, which is not a good answer.

Let's try some rearranging:

top and bottom by the conjugate of the top:

$$\frac{2 - \sqrt{x}}{4 - x} \times \frac{2 + \sqrt{x}}{2 + \sqrt{x}}$$

simplify top using $(a + b)(a - b) = a^2 - b^2$:

$$\frac{2^2 - (\sqrt{x})^2}{(4 - x)(2 + \sqrt{x})}$$

Simplify top further:

$$\frac{(4 - x)}{(4 - x)(2 + \sqrt{x})}$$

Cancel $(4-x)$ from top and bottom:

$$\frac{1}{2 + \sqrt{x}}$$

$$\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{4 - x} = \lim_{x \rightarrow 4} \frac{1}{2 + \sqrt{x}} = \frac{1}{2 + \sqrt{4}} = \frac{1}{4}$$

You try!

$$\lim_{x \rightarrow 1} \frac{2 - \sqrt{3 + x}}{x - 1}$$

Get Common Denominator

$$\lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$$

Multiply denominators 4 and

$$\lim_{x \rightarrow -4} \frac{\frac{x}{4x} + \frac{4}{4x}}{4 + x} = \lim_{x \rightarrow -4} \frac{\frac{x+4}{4x}}{4+x} = \lim_{x \rightarrow -4} \frac{x+4}{4x} \cdot \frac{1}{x+4}$$

$$= \lim_{x \rightarrow -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$$

You try!

Evaluate $\lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$.







