Rolles's Theorem and Mean Value Theorem Practice

Show all work. No calculator unless otherwise stated.

Multiple Choice

_____ 1. Determine if the function $f(x)=x\sqrt{6-x}$ satisfies the hypothesis of Rolle's Theorem on the interval $[0,6]$, and if it does, find all numbers c satisfying the conclusion of that theorem.

(A) 2, 3 (B) 4, 5 (C) 5 (D) 4 (E) hypothesis not satisfied

____ 2. Let f be a function defined on $[-1,1]$ such that $f(-1)=f(1)$. Consider the following properties that f might have:

I. f is continuous on $[-1,1]$, differentiable on $(-1,1)$.

II. $f(x)=\cos^3 x$

III. $f(x)=|\sin \pi x|$

Which properties ensure that there exists a c in $(-1,1)$ at which $f'(c)=0$?

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

_____ 3. Determine if the function $f(x)=x^3-x-1$ satisfies the hypothesis of the MVT on $[-1,2]$. If it does, find all possible values of c satisfying the conclusion of the MVT.

(A) $-\frac{1}{2}$

(B) $-1,\ 1$

(C) 0

(D) 1

(E) hypothesis not satisfied

_____ 4. Determine if the function $f(x)=x+x^{2/3}(1-x)^{1/3}$ satisfies the hypothesis of the MVT on $[0,1]$. If it does, find all possible values of c satisfying the conclusion of the MVT. (You will have to factor out least powers.)

(A) $\frac{2}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

(E) hypothesis not satisfied

_____ 5. Which of the following functions below satisfy the hypothesis of the MVT?

I. $f(x)=\frac{1}{x+1}$ on $[0,2]$

II. $f(x)=x^{1/3}$ on $[0,1]$

III. $f(x)=|x|$ on $[-1,1]$

(A) I only (B) I and II only (C) I and III only (D) II only (E) II and III only

_____ 6. As a graduation present, Jenna received a sports car which she drives very fast but very, very smoothly and safely. She always covers the 53 miles from her apartment in Austin, Texas to her parents' home in New Braunfels in less than 48 minutes. To slow her down, her dad decides to change the speed limit (he has connections.) Which one of the speed limits below is the highest speed her father can post, but still catch her speeding at some point on her trip?

(A) 55 mph (B) 70 mph (C) 65 mph (D) 50 mph (E) 60 mph

_____ 7. Consider the following statements:

I. $f(x)$ is continuous on $[a,b]$

II. $f(x)$ is differentiable on (a,b)

III. $f(a)=f(b)$

Which of the above statements are required in order to guarantee a $c\in(a,b)$ such that $f'(c)(b-a)=f(b)-f(a)$?

(A) I only (B) I and II only (C) I, II, and III (D) III only (E) I and III only

8. Without looking at your notes, state the Mean Value Theorem.

If . . .

then . . .

9. Determine if Rolle's Theorem can be applied to the following functions on the given interval. If so, find the value(s) guaranteed by the theorem.

(a) $f(x)=\cos 2x$ on $\left[-\frac{\pi}{12},\frac{\pi}{6}\right]$

(b) $g(x)=\begin{cases} x, & 0\le x\le \frac{1}{2} \\ 1-x, & \frac{1}{2}<x\le 1 \end{cases}$ on $[0,1]$

10. Determine if the MVT can be applied to the following functions on the given interval. If so, find the exact value(s) guaranteed by the theorem. Be sure to show your set up in finding the value(s).

(a) $f(x)=\ln(x-1)$ on $[2,4]$

(b) $f(x)=\begin{cases} \arcsin x, & -1\le x<1 \\ \frac{x}{2}, & 1\le x\le 3 \end{cases}$ on $[-1,3]$.

(c) $g(x)=\frac{x+1}{x}$ on $\left[\frac{1}{2},2\right]$

(d) $f(x)=2\sin x+\sin 2x$ on $[0,\pi]$