1. $$\int_0^1 \frac{10\sqrt{x}}{\left(1+x^{3/2}\right)^2}\,dx$$

$u = 1 + x^{3/2}$

$du = \frac{3}{2} x^{1/2}$

$\frac{2}{3} du = x^{1/2}$

$$= \int_0^1 \frac{10\,x^{1/2}}{\left(1+x^{3/2}\right)^2}\,dx$$

$$= \frac{2}{3} 10 \int_0^1 \frac{1}{u^2}\,du$$

$$= \frac{20}{3} \int_0^1 u^{-2}\,du$$

$$= -\frac{20}{3}\left(\frac{1}{1+x^{3/2}}\right)\Bigg|_0$$

$$= \frac{-20}{3}\left[\left(\frac{1}{1+1}\right) - \left(\frac{1}{1+0}\right)\right]$$

$$= \frac{-20}{3}\left[\frac{1}{2} - 1\right]$$

$$= \frac{-20}{6} + \frac{20}{3}$$

$\int_{-\pi} \sqrt{4+3\sin x}$

$$= \frac{1}{3}\int_{-\pi}^{\pi} \frac{1}{\sqrt{u}}\,du$$

$$u = 4+3\sin x$$

$$du = 3\cos x$$

$$\frac{1}{3}du = \cos x$$

$$= \frac{1}{3}\int_{-\pi}^{\pi} \frac{1}{u^{\frac{1}{2}}}\,du$$

$$= \frac{1}{3}\int_{-\pi}^{\pi} u^{-\frac{1}{2}}\,du$$

$$= 2\cdot\frac{1}{3}u^{\frac{1}{2}}$$

$$= \frac{2}{3}(4+3\sin x)\Big|_{-\pi}^{\pi}$$

$$= \frac{2}{3}\left[(4+3\sin\pi)-(4+3\sin-\pi)\right]$$

$$= \frac{2}{3}\left[\cancel{4-4}\right] = 0$$

$\frac{dy}{dx} = (y+5)(x+2), \quad y = 1$ when $x = 0$

$$\int \frac{1}{y+5}\,dy = \int (x+2)\,dx$$

$$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$$

$$e \qquad e$$

$$y+5 = e^{\frac{1}{2}x^2} \cdot e^{2x} \cdot e^{C}$$

$$y = Ce^{\frac{1}{2}x^2} \cdot e^{2x} - 5$$

$$(1) = Ce^{0} \cdot e^{0} - 5$$

$du = 1\,dx$

$$1 = C - 5$$

$$6 = C$$

$$\boxed{y = 5e^{\frac{1}{2}x^2} \cdot e^{2x} - 5}$$

$$\frac{dy}{dx}=(\cos x)e^{y+\sin x},\quad y=0 \text{ when } x=0$$

$u=\sin x$

$du=\cos x\,dx$

$$\int e^{-y}\,dy=\int \cos x\, e^{\sin x}\,dx$$

$$-e^{-y}=e^{\sin x}+c$$

$$\ln(e^{-y})=\ln(-e^{\sin x}+c)$$

$$-y=\ln|-e^{\sin x}+c|$$

$$0=\ln|-e^{\sin 0}+c|$$

$$0=\ln|-1+c|$$

$$e \quad e$$

$$1=-1+c$$

$$2=c$$

$$y=-\ln|-e^{\sin x}+2|$$

$$\int \frac{2x+16}{x^2+x-6}\,dx = \int \frac{2x+16}{(x+3)(x-2)}\,dx$$

$$= -2\int \frac{1}{x+3} + 4\int \frac{1}{x-2}\,dx$$

$\boxed{-3}$ $\quad \frac{2(-3)+16}{-3-2} = \frac{10}{-5} = -2$

$$= -2\ln|x+3| + 4\ln|x-2| + C$$

$\boxed{2}$ $\quad \frac{2(2)+16}{(2)+3} = \frac{20}{5} = 4$

$$= 4\ln|x-2| - 2\ln|x+3| + C$$

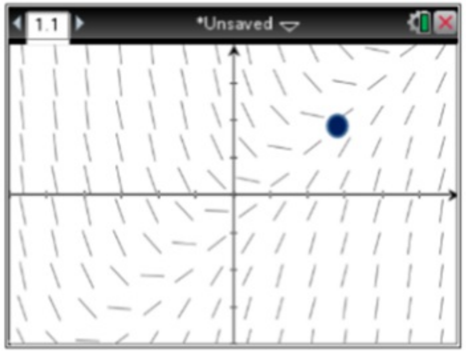
$$= \ln\frac{|x-2|^4}{|x+3|^2} + C$$

$$\int 10xe^{5x}\,dx$$

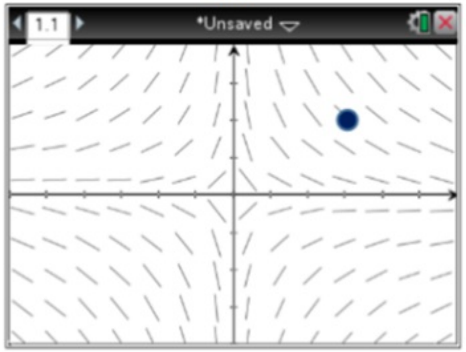
u	dv	$+/-$
$10x$	e^{5x}	$+$
10	$\frac{1}{5}e^{5x}$	$-$
0	$\frac{1}{25}e^{5x}$	$+$

$$= \frac{10}{5}xe^{5x} - \frac{10}{25}e^{5x} + C$$

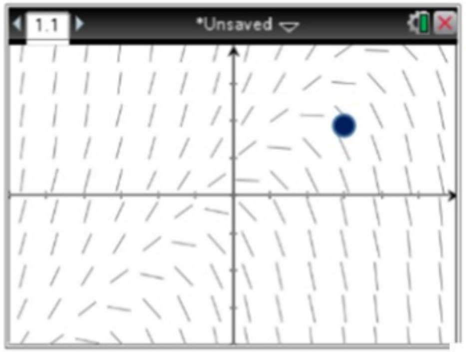
(A)



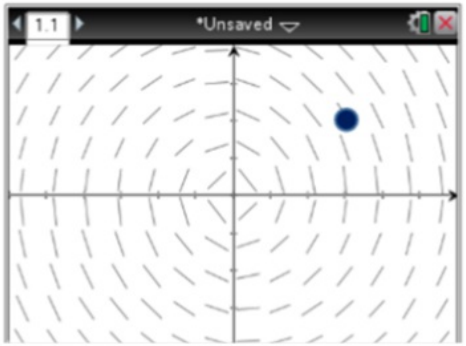
(B)



(C)



(D)



7. $\dfrac{dy}{dx} = x - y$ A

8. $\dfrac{dy}{dx} = y - x$ C

9. $\dfrac{dy}{dx} = -\dfrac{y}{x}$ B

10. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ D

$\int x^4 \cos 2x \, dx$ (Use tabular method)

u	dv	+/−
x^4	$\cos 2x$	+
$4x^3$	$\frac{1}{2}\sin 2x$	−
$12x^2$	$-\frac{1}{4}\cos 2x$	+
$24x$	$-\frac{1}{8}\sin 2x$	−
24	$\frac{1}{16}\cos 2x$	+
0	$\frac{1}{32}\sin 2x$	−

$$= \frac{1}{2}x^4 \sin 2x + \frac{4}{4}x^3 \cos 2x - \frac{12}{8}x^2 \sin 2x - \frac{24}{16}x \cos 2x + \frac{24}{32}\sin 2x + C$$

$\tan x = \frac{\quad}{\cos x}$

$$\int x \sec^2 x \, dx$$

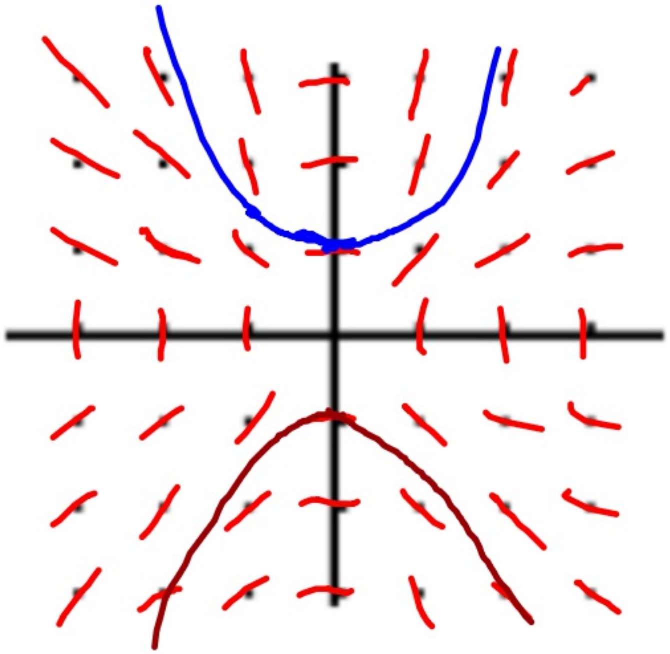
u	dv	$+/-$
x	$\sec^2 x$	$+$
1	$\tan x$	$-$
0	$-\ln\lvert\cos x\rvert$	$+$

$$= x\tan x + \ln\lvert\cos x\rvert + C$$

1 5,

Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

A) On the axes provided, sketch a slope field for the given differential equation.



(0,0) =

(0,1) = 0

(0,2) = 0

(0,3) = 0

(1,1) = 1

(2,2) = 1

(1,0) = und

(2,0) = und

(3,0) = und

(4,0) = und

b. (0,1)

d. (0,-1)

C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

D) Sketch a solution curve that passes through the point (0,-1) on your slope field.

$$y^2 = x^2 + C$$

$$y = \sqrt{\cancel{x}^2 + C}$$

$$1 = \sqrt{0^2 + C}$$

$$1^2 = \sqrt{C}^2$$

$$\boxed{y = x^2 + 1}$$

14.

Given the **logistic differential equation** $\frac{dA}{dt} = A\left(20 - \frac{A}{4}\right)$, where $A(0) = 15$, what is $\lim_{t\to\infty} A(t)$?

$= \frac{1}{4}A(80 - A)$

(A) 20
(B) 40
(C) 60
(D) 80
(E) 100

$L = 80$ carrying capacity