Name_____ Date_____ Period_____

Worksheet 7.2—Parametric & Vector Accumulation
Show all work. No calculator except unless specifically stated.

Short Answer/Free Response

1. If $x = e^{2t}$ and $y = \sin(3t)$, find $\frac{dy}{dx}$ in terms of t.

2. Write an integral expression to represent the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^2 t$ for $0 \le t \le \frac{\pi}{2}$.

3. For what value(s) of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

4. Find the equation of the tangent line to the curve given by the parametric equations $x(t) = 3t^2 - 4t + 2$ and $y(t) = t^3 - 4t$ at the point on the curve where $t = 1$.

5. If $x(t) = e^t + 1$ and $y = 2e^{2t}$ are the equations of the path of a particle moving in the $xy-$plane, write an equation for the path of the particle in terms of x and y.

6. (Calculator) A particle moves in the $xy-$plane so that its position at any time t is given by $x = \cos(5t)$ and $y = t^3$. What is the speed of the particle when $t = 2$?

7. (Calculator) The position of a particle at time $t \geq 0$ is given by the parametric equations $x(t) = \dfrac{(t-2)^3}{3} + 4$ and $y(t) = t^2 - 4t + 4$.

(a) Find the magnitude of the velocity vector at $t = 1$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.

(c) When is the particle at rest? What is its position at that time?

8. (Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ at time $t \geq 0$ with $\frac{dx}{dt} = 1 + \tan\left(t^2\right)$ and $\frac{dy}{dt} = 3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t = 5$.

9. (Calculator) A particle moves in the $xy-$plane so that the position of the particle is given by $x(t) = t + \cos t$ and $y(t) = 3t + 2\sin t$, $0 \leq t \leq \pi$. Find the velocity vector when the particle's vertical position is $y = 5$.

10. (Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ at time t with $\frac{dx}{dt} = 2\sin\left(t^3\right)$ and $\frac{dy}{dt} = \cos\left(t^2\right)$ for $0 \le t \le 4$. At time $t = 1$, the object is at the position $(3, 4)$.

(a) Write an equation for the line tangent to the curve at $(3, 4)$.

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 2$.

Multiple Choice:

11. (Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ with $\frac{dx}{dt} = \cos\left(t^2\right)$ and $\frac{dy}{dt} = \sin\left(t^3\right)$. At time $t = 0$, the object is at position $(4, 7)$. Where is the particle when $t = 2$?

(A) $\langle -0.564, 0.989 \rangle$ (B) $\langle 0.461, 0.452 \rangle$ (C) $\langle 3.346, 7.989 \rangle$
(D) $\langle 4.461, 7.452 \rangle$ (E) $\langle 5.962, 8.962 \rangle$

12. (Calculator) The path of a particle moving in the plane is defined parametrically as a function of time t by $x = \sin 2t$ and $y = \cos 5t$. What is the speed of the particle at $t = 2$?

(A) 1.130 (B) 3.018 (C) $\langle -1.307, 2.720 \rangle$ (D) $\langle 0.757, 0.839 \rangle$ (E) $\langle 1.307, 2.720 \rangle$

13. For what values of *t* does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only (B) 1 only (C) 0 and 2/3 only (D) 0, 2/3, and 1 (E) No value

14. The distance traveled by a particle from $t = 0$ to $t = 4$ whose position is given by the vector $\bar{s}(t) = \langle t^2, t \rangle$ is given by

(A) $\int_0^4 \sqrt{4t+1}\,dt$ (B) $2\int_0^4 \sqrt{t^2+1}\,dt$ (C) $\int_0^4 \sqrt{2t^2+1}\,dt$ (D) $\int_0^4 \sqrt{4t^2+1}\,dt$ (E) $2\pi\int_0^4 \sqrt{4t^2+1}\,dt$